~~Pitfalls of N-R method~~

Fixed Point iteration

express $x = g(x)$

iterate.

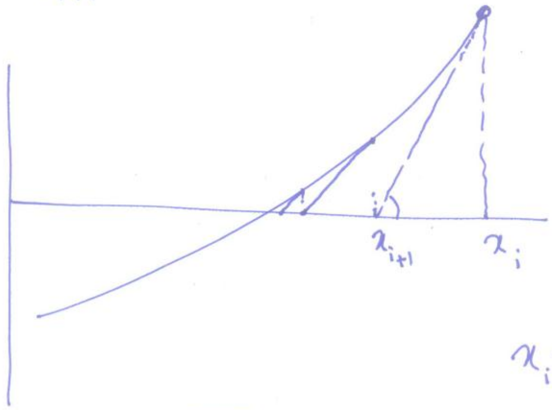
Secant Method

1. Evaluation of the derivative
2. Approximate

$$f'(x_i) \simeq \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)\left[x_{i-1} - x_i\right]}{f(x_{i-1}) - f(x_i)}$$

NR: $x_{i+1} =$

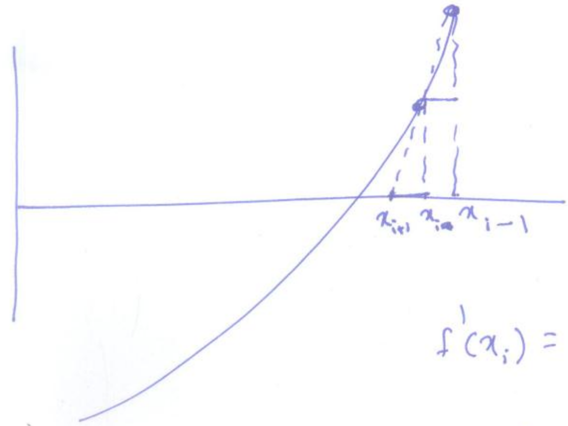


$$x_i - x_{i+1} = \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$x_{i+1} =$



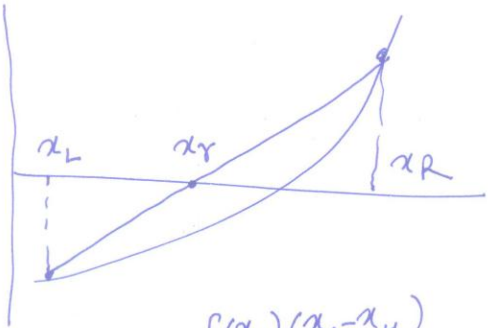
$$\frac{x_{i-1} - x_i}{x_i - x_{i+1}} = \frac{f(x_{i-1}) - f(x_i)}{f(x_i)}$$

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_i - \frac{f(x_i)\,(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$$x_{i+1} = x_i - \frac{f(x_i)\{x_i - x_{i-1}\}}{f(x_i) - f(x_{i-1})}$$

False position



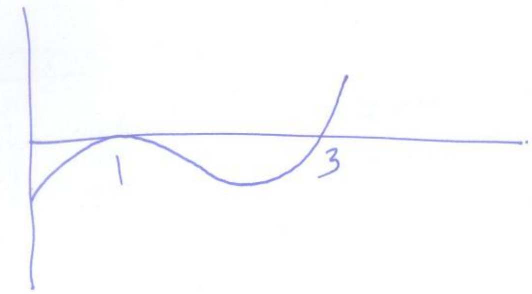
$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

Modified secant Method

use a small purturbation around x_i

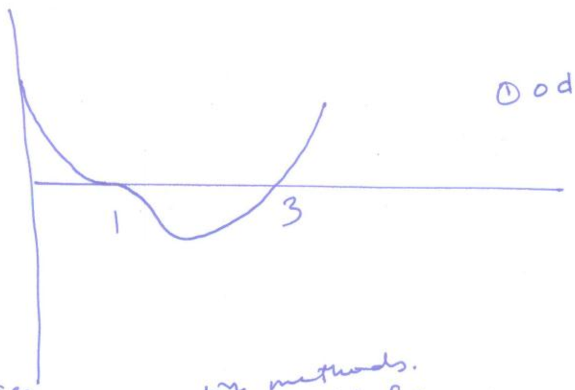
Multiple roots

$$f(x) = x^3 - 5x^2 + 7x - 3$$
$$= (x-3)(x-1)(x-1)$$



even multiple roots

$$f(x) = (x-3)(x-1)(x-1)(x-1)$$



① odd multiple roots

Difficulties

i. cannot use bracketing methods.

difficult for

$$\left.\begin{array}{l} f(x) = 0 \\ f'(x) = 0 \end{array}\right\}$$ N-R secant.

$f(x)$ will always reach zero before $f'(x)$

Incorporate the zero check for $f(x)$ before $f'(x)$.

... zero.

It yields 2 roots.
Sign is chosen to agree with sign of b {Largest denominator}

repeat.

Strategy

a - if only real roots are being located
choose two original points that are
nearest the new root.

b - if both real and complex roots are
being evaluated a sequential approach
is employed.
x_1, x_2, x_3

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$m =$ multiplicity of the r...

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Fixed Point iteration

$$u(x,y) = x^2 + xy - 10 = 0$$

$$v(x,y) = y + 3xy^2 - 57 = 0$$

$$x_{i+1} = \frac{10 - x_i^2}{y_i}$$

$$y_{i+1} = 57 - 3x_i y_i^2$$

$$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$$

f_1

f_2

f_3

$$\boxed{|g'(x)| < 1}$$

$$\left|\frac{\partial u}{\partial x}\right| + \left|\frac{\partial u}{\partial y}\right| < 1$$

$$\left|\frac{\partial v}{\partial x}\right| + \left|\frac{\partial v}{\partial y}\right| < 1$$

$$x_{i+1} = x_i - \frac{u_i}{\quad}$$